The M.K.S. System of Units Applied to Electroacoustics

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Purpose and Scope

The subject of electroacoustics deals with the transformation of electric energy to sound energy and *vice versa* by means of various electroacoustic transducers—microphones and loudspeakers. The quantitative investigation of such transfers requires a definite system of units for expressing the magnitudes involved. At present two sets of units are commonly employed; electric quantities are expressed in the international series of practical electric units, the *volt*, *ohm*, *ampere*, *joule*, etc.; while mechanic quantities are stated in the *centimeter*, *gram*, *second* (c.g.s.), system. These two sets of units are not homogeneous, one with the other, and their coincident use in electroacoustic work makes necessary the employment of unwieldy transformation coefficients. The *joule*, unit of energy in the practical series, is equivalent to 10^7 *ergs* of the c.g.s. system; the practical unit of power, the *watt*, is equal to 10^7 *ergs/second*, etc.

A great saving in labor on the part of electroacoustic workers would be effected if a system of mechanic units, homogeneous with the practical electric unit series, were employed. Such a system, the Giorgi *meter*, *kilogram*, *second* (m.k.s.) system of units, is available for use and its recent adoption by the I.E.C. (International Electrotechnical Commission) endows it with sufficient importance to warrant its immediate application.[1] It is the purpose of this paper to express commonly used acoustic quantities in the m.k.s. system, to note their magnitudes relative to the corresponding c.g.s. units, and to point out the advantages thereby obtained.

Acoustic Units

The unit of length in the m.k.s. system is the *meter* (*m*) ($=10^2$ *cm*), the unit of mass is the *kilogram* (*kg*) ($=10^3$ *gm*). Time is measured in terms of the *mean solar second* (*s*) in both systems.

Sound wave-length is measured in *meters* (*m*) ($=10^2$ *cm*). The wave-length of a 1000 *c.p.s.* sound wave in air, at ordinary barometric pressure intensity and temperature, is 0.344 (*m*).

The velocity of propagation of a sound wave is stated in *meters/second* (*m/s*) ($=10^2$ *cm/s*). A free sound wave in air, under normal conditions, (20°C, 1 *standard atmosphere*) is propagated at a velocity, 344 (*m/s*). (This unit is almost universally used at the present time). Particle velocity is likewise expressed in (*m/s*).

Sound pressure (intensity) is measured in *newtons/square meter* (*newtons/m*^{2}) ($=10$ *barye* or *dynes/cm*^{2}) (*Newton* is the tentative name for the m.k.s. unit of force, one *joule/meter*, equals 10^5 *dynes*). The present c.g.s. reference pressure (0.2) *millibaryed*) is 2×10^{-5} *newton/m*^{2} or 20 *micronewtons/m*^{2}. Normal barometric pressure is 1.0132×10^5 *newtons/m*^{2} $=0.10132$ *mega-newton/m*^{2}.

The unit of sound energy flow is the *watt* ($=10^7$ *ergs/s*). For an effective sound pressure, p (*newtons/m*^{2}) in a plane or spherical free wave propagated with a velocity, c (*m/s*), in a gas of density, ρ (*kg/m*^{3}), the sound energy flow, P (*watts*), through an area, a (m^2), is:

$$\mathrm{P}=(p^2a/\rho c)\cos\theta \quad \textit{watts},$$

where θ is the angle of incidence of the sound wave on the area, a.

Sound intensity, the sound energy passing through unit area in unit time, is expressed in *watts/square meter* (*watts/m*^{2}) ($=10^3$ *ergs/s·cm*^{2}). For an effective pressure, p (*newtons/m*^{2}), in a plane or spherical free wave propagated with a velocity, c (*m/s*), in a gas of density, ρ (*kg/m*^{3}), the sound intensity, I (*watts/m*^{2}) in the direction of propagation may be written:

$$\mathrm{I}=(p^2/\rho c) \quad \textit{watts/m}^2.$$

Sound energy density has the units of *joules/cubic meter* (*joule/m*^{3}) ($=10^{+1}$ *ergs/cm*^{3}).

[1] A. E. Kennelly, "I.E.C. Adopts M.K.S. System of Units," Elec. Eng. Dec. (1935). I.E.C. Document R.M. 118. Minutes of the I.E.C., E.M.M.U. Meeting, Scheveningen-Brussels, June (1935). Gino Sacerdote, "L'Applicazione Delle Unità M.K.S. Elettromagnetiche (Giorgi) Nel Campo Dell'Elettroacoustica," Alta Frequenza **9**, 570 (1936).

Acoustic Impedance

The subject of acoustic impedance is confused by the existence in the literature of conflicting definitions of this important quantity. Crandall, Kennelly, Morse, and others define the acoustic impedance of a sound medium to a surface lying in a wave front as the product of the surface area by the ratio of sound pressure to particle velocity in the medium.

$$Z=(p/q)a.$$

From this definition it follows logically that the motion of a given piston in a sound medium is impeded more than is the motion of a similar piston having a smaller surface area. Furthermore the dimensions of acoustic impedance according to this definition, are those of a force divided by a velocity, identical with the dimensions of mechanic impedance.

$$Z=(p/q)a,$$

$$Z=\frac{[\text{Force/Area}]}{[\text{Velocity}]}[\text{Area}],$$

$$Z=[\text{Force}]/[\text{Velocity}].$$

Therefore in determining the behavior of a mechanic system in a sound medium (a vibrating piston in air) the acoustic impedance offered by the medium introduces a directly additive modification of the mechanic impedance of the system. In short, by this definition, acoustic impedance *is* mechanic impedance.[2]

The fundamental dimensions of this kind of acoustic impedance are a mass divided by a time.

$$Z=\frac{[\text{Force}]}{[\text{Velocity}]}=\frac{[\text{MLT}^{-2}]}{[\text{LT}^{-1}]}=[\text{MT}^{-1}].$$

The unit in the m.k.s. system is the *kilogram/second* (*kg/s*) ($=10^3$ *gm/s*). It would be advantageous if the name *mechanic ohm* should be used for the unit of mechanic *and* acoustic impedance having this magnitude.

The tentative "American Standards for Acoustical Terminology" define the acoustic impedance of a sound medium on a given surface lying in a wave front as "the complex quotient of the sound pressure (force per unit area) on that surface by the flux (volume velocity or linear velocity multiplied by the area) through the surface."[3]

$$Z=(p/q)(1/a).$$

According to this definition the motion of a given piston in a sound medium would be less impeded than would be the motion of a piston having a smaller surface area. However a definition of an acoustic impedance of this type is of use in problems in which the actual flow of sound energy through orifices is dealt with and a concept of impedance somewhat analogous to the resistance of a conductor to electric current flow is needed.[4]

The ambiguity introduced by the above duality of definition does not affect the concept of "specific acoustic impedance," defined for a plane sound wave as the ratio of the sound pressure to the particle velocity in the sound medium.

$$Z_s=p/q.$$

The unit in the m.k.s. system is the *kilogram/second square meter* ($kg/s\cdot m^2$) ($=10^{-1}$ $gm/s\cdot cm^2$ (Suggested name: *mechanic ohm per square meter*).

For the case of a plane sound wave propagated with a velocity, c (m/s), in a sound medium of density, ρ (kg/m), the specific acoustic impedance reduces very simply to:

$$Z_s=\rho c \quad kg/s\cdot m^2.$$

The density of air under normal pressure at 20°C

[2] A. E. Kennelly and J. H. Cook, "Mechanic Impedance in M.K.S. Units." Elec. Eng. **56**, 1062–63 (1937).

[3] American Tentative Standard—Acoustical Terminology 24.1 (1936). American Standards Association.

[4] It is evident that these two definitions of acoustic impedance differ because they deal with two entirely different concepts; one the impedance offered to a plate vibrating in a sound medium, the other the impedance to the flow of fluid offered by an orifice in a sound medium. These two concepts might best be discriminated between by the use of different unit names such as *piston impedance* and *orfice impedance*, Z_p and Z_a, respectively. If this latter definition (Z_a) is used exclusively for "acoustic impedance" it is necessary, when studying the motion of a vibrating piston in a sound medium, to multiply the acoustic impedance of the system by the square of the piston area in order that the contribution of the sound medium to the mechanic impedance of the system may be determined. The fundamental dimensions of acoustic impedance as defined in the "Tentative Standards" are $[\text{ML}^{-4}\text{T}^{-1}]$.

$$Z=\frac{p}{q}\frac{1}{a}=\frac{[\text{MLT}^{-2}/\text{L}^2]}{[\text{LT}^{-1}]}\frac{1}{[\text{L}^2]}=[\text{ML}^{-4}\text{T}^{-1}].$$

The unit in the m.k.s. system is the *kilogram/second-meter-to-the-fourth-power* ($kg/s\cdot m^4$) ($=10^{-5}$ $gm/s\cdot cm^4$).

is 1.205 kg/m^3; the velocity of propagation of a sound wave in air under these conditions is 344.0 m/s. Thus:

$Z_s = 415$ *mech. ohms*/m^2 ($kg/s \cdot m^2$) for air at 20°C, 1 *atmosphere*.

Architectural Acoustics

Reverberation time is measured in terms of the *second* (*s*), a unit common to both systems. Sabine's equation for the reverberation time (T), in which the energy density is reduced by a factor 10^6, is

$$T = \frac{0.16\ V}{a\bar{\alpha}}\ (s),$$

where V is the volume (m^3) of a room having a total surface area (walls, floor and ceiling) (m^2), with an average absorption coefficient, $\bar{\alpha}$.

The steady state energy density, E(*joules*/m^3), in this room when a sound source, P (*watts*), is operating in it is:

$$E = \frac{4P}{c\bar{\alpha}a}\ joules/m^3.$$

The coefficients of viscosity and of kinematic viscosity of a sound medium are used in studies of absorbing materials and of sound transmission. The coefficient of viscosity is measured in units of *kilograms/meter-second* ($kg/m \cdot s$) ($=10^1\ gm/cm \cdot s$). The coefficient of kinematic viscosity has the unit, *square meter/second* (m^2/s) ($=10^4\ cm^2/s$). For air at ordinary temperature and pressure:

Coefficient of viscosity $=18.4 \times 10^{-6}\ kg/m \cdot s$
Coefficient of kinematic viscosity $=1.32 \times 10^{-5}\ m^2/s$.

Conclusions

Electroacoustics embodies two branches of applied science, namely, (1) mechanics, (2) electrics including electromagnetics, electrostatics, piezoelectrics and thermoelectrics.

If mechanic quantities are expressed in c.g.s. units it is also necessary to measure electromagnetic quantities in c.g.s. electromagnetic units if homogeneity is to be maintained. It is however, almost impossible to avoid using the practical series of electric units (*volt*, *ohm*, *ampere*, etc.) because these are invariably used in electric measurements as well as in commercial specifications and descriptions. This fact leads to the use of a dual system of units, mechanic measures being made in the c.g.s. system, electromagnetic magnitudes being expressed in the practical series. These two systems are not homogeneous, a given quantity having units of different sizes in the two systems, (e.g. the unit of energy in the c.g.s. system is the *erg* and is equal to 10^{-7} *joule*, the m.k.s. unit of energy). Thus the use of dual systems makes necessary unwieldy transformation coefficients that obscure the physical facts being investigated and complicate the numerical calculations.

When, however, the m.k.s. unit system is used for both mechanic and electric quantities all transformation coefficients disappear.

A further advantage of m.k.s. mechanic units is their size which is suitable for engineering work.[5]

At least one textbook[6] has thus far been published using the m.k.s. system of units exclusively, and several others are in preparation.

[5] A. E. Kennelly, "The M.K.S. System of Giorgi as Adopted by the International Electrotechnical Commission (I.E.C.) in June 1935," J. Eng. Ed. Dec. (1936).

[6] P. Vigoureux and C. E. Webb, *Principles of Electric and Magnetic Measurements* (Blackie & Son, Ltd., Glasgow; Prentice-Hall, Inc., New York, 1936).

APPENDIX I

TABLE I. *A comparative table of mechanic, acoustic, and electric units of use in electroacoustics.*

QUANTITY	SYMBOL	m.k.s. UNIT	c.g.s. UNIT	c.g.s. UNITS IN 1 m.k.s. UNIT	m.k.s. UNITS IN 1 c.g.s. UNIT
Mass	M, m	kilogram, kg	gram, gm	10^3	10^{-3}
Length	L, l	meter, m	centimeter, cm	10^2	10^{-2}
Time	T, t	second	second	1	1
Density		kilogram per cu. meter kg/m³	gram per cu. centimeter, gm/cm³	10^{-3}	10^3
Velocity	V, v	meters per second, m/s	centimeters per second, cm/s	10^2	10^{-2}
Force	F, f	newton, kg·m/s²	dyne, gm·cm/s²	10^5	10^{-5}
Pressure (intensity)	p	kg/s·m	barye, gm/s·cm	10^1	10^{-1}
Sound Energy Flow	P*	watt	erg/s	10^7	10^{-7}
Sound Intensity	I	watt/m²	erg/s·cm²	10^3	10^{-3}
Sound Energy Density	E	joule/m³	erg/cm³	10^1	10^{-1}
"Diaphragm" Acoustic Impedance	Z_p	m.k.s. mechanic ohm, kg/s	c.g.s. mechanic ohm, gm/s	10^3	10^{-3}
"Orifice" Acoustic Impedance	Z_a	kg/s·m⁴	acoustic ohm, gm/s·cm⁴	10^{-5}	10^5
"Specific" Acoustic Impedance	Z_s	kg/s·m²	gm/s·cm²	10^{-1}	10^1
Viscosity		kg/s·m	poise, gm/s·cm	10^1	10^{-1}
Kinematic Viscosity		m²/s	cm²/s	10^4	10^{-4}
Work	W	joule	erg	10^7	10^{-7}
Power	P, P	watt	erg/s	10^7	10^{-7}
Elastic Coefficient	S	newton/m	dyne/cm	10^3	10^{-3}
Mechanic Impedance	Z	mechanic ohm, kg/s	mechanic abohm, gm/s	10^3	10^{-3}
Force Factor	$\mathcal{A}$	newton/ampere	dyne/abampere	10^6	10^{-6}
Electric Current	I, i	ampere	abampere	10^{-1}	10^1
Electric Resistance	R	ohm	abohm	10^9	10^{-9}
Electric Inductance	L	henry	abhenry	10^9	10^{-9}
Magnetomotive Force	$\mathfrak{F}$	ampere turn		$4\pi\times10^{-1}$	$\frac{1}{4\pi}\times10^{+1}$
Magnetic Intensity	$\mathcal{H}$	ampere-turn per meter	oersted	$4\pi\times10^{-2}$	$\frac{1}{4\pi}\times10^2$
Magnetic Flux Density	$\mathcal{B}$	weber/square meter	gauss	10^4	10^{-4}
Magnetic Reluctance	$\mathcal{R}$	ampere-turn/weber		$4\pi\times10^{-9}$	$\frac{1}{4\pi}\times10^9$
Magnetic Permanence	$\mathcal{P}$	weber/ampere-turn		$\frac{1}{4\pi}\times10^9$	$4\pi\times10^{-9}$

* J is sometimes used for this quantity.

APPENDIX II

An electroacoustic problem illustrating the use of the m.k.s. system of units

The diaphragm of an electrodynamic loudspeaker is assumed to act as a rigid piston. We shall consider the behavior of this device operating first in a vacuum and secondly in air. We shall further subdivide each of these cases into operation with an applied mechanic vibromotive force and operation under the influence of an alternating electromotive force applied to the voice coil terminals inducing a mechanic vibromotive force. Mechanic friction in the diaphragm and its supports is neglected in the analysis. The following constants of the loudspeaker are assumed:

Radius of diaphragm	$0.1m$
Area of diaphragm	$a=0.01\pi\ m^2$
Mass of diaphragm (including attached voice coil)	$M=5\times10^{-3}\ kg$
Elastance of diaphragm support	$S=5\times10^2\ newton/m$
Force factor of electromagnetic drive	$\mathcal{A}=2\ newton/amp.$
Electric resistance of voice coil	$R_c=20\ ohms$
Inductance of voice coil	$L_c=10^{-2}\ henrys$

Case I_a Loudspeaker in a vacuum
Mechanic vibromotive force applied
F = 2 *newtons* r.m.s.

The mechanic impedance of the system is:

$$Z_a=0+j(M\omega-S/\omega)\ \ mech.\ ohms\ (kg/s).$$

With an r.m.s. force, F *newtons*, applied the r.m.s. velocity of the diaphragm is:

$$V=F/Z\ \ m/s.$$

This system is purely reactive, no active power being absorbed from the driving agency. (See Table II.)

We must first determine the impedance to electric current flow offered by the moving coil. This impedance includes the electric impedance of the coil, when stationary, and the motional electric impedance of the device. The motional electric impedance is given by:

$$Z_M=\mathcal{A}^2/Z\ \ ohms.$$

The electric impedance of the voice coil, when stationary, is:

$$Z_c=R_c+jL_c\omega\ \ ohms.$$

The total electric impedance of the moving voice coil is:

$$Z_E=Z_M+Z_c\ \ ohms.$$

TABLE II.

Case Ia. Loudspeaker in a vacuum.
Mechanic vibromotive force applied; F = 2 *newtons* r.m.s.

ω	Z	V
rad/s	*mech. ohms = kg/s*	*m/s*
100	$-j4.5$	$j.445$
316	0	∞
1000	$j4.5$	$-j.445$
10000	$j50$	$-j.040$

Case Ib. Speaker in a vacuum.
Alternating electromotive force applied to voice coil $E = 10$ *volts* r.m.s.

TABLE III.

Case Ib. Loudspeaker in a vacuum.
Alternating e.m.f. applied to voice coil $E = 10$ *volts* r.m.s.

ω *rad/s*	Z_c *ohms*	Z_M *ohms*	Z_E *ohms*
100	$20+j1.0$	$j.89$	$20+j1.89$
316	$20+j3.16$	∞	∞
1000	$20+j10.0$	$-j.89$	$20+j9.11$
10000	$20+j100.0$	$-j.08$	$20+j100$

I *amp.*	F *newtons*	V *m/s*
$0.496-j.047$	$0.992-j.094$	$0.021 +j.221$
0	0	—
$.414-j.089$	$.828-j.178$	$-.0396-j.184$
$.019-j.096$	$.038-j.192$	$-.0038-j.00076$

Case IIa. Loudspeaker in air.
Mechanic vibromotive force applied F = 2 *newtons* r.m.s.

TABLE IV.

Case IIa. Loudspeaker in air.
Mechanic vibromotive force applied F = 2 *newtons* r.m.s.

ω *rad/s*	Z *mech. ohms = kg/s*	V *m/s*	P *watts*
100	$13.0-j4.5$	$0.137 +j.048$	0.272
316	13.0	.154	.308
1000	$13.0+j4.5$	$.137 -j.048$	.272
10000	$13.0+j50$	$.0097-j.0375$	.0195

Case IIb. Loudspeaker in air.
Alternating electromotive force applied to voice coil $E = 10$ *volts* r.m.s.

Hence the electric current that flows in the voice coil may be calculated:

$$I = E/Z_E \ \textit{amp.}$$

TABLE V.

Case IIb. Loudspeaker in air.
Alternating e.m.f. applied to voice coil $E = 10$ *volts* r.m.s.

ω *rad/s*	Z *mech ohms = kg/s*	Z_M *ohms*	Z_E *ohms*	I *amp*
100	$13.0-j4.5$	$0.275 +j.095$	$20.275 +j1.095$	$0.491 -j.0267$
316	13.0	.308	$20.308 +j3.16$	$.480 -j.0748$
1000	$13.0+j4.5$	$.275 -j.095$	$20.275 +j9.905$	$.398 -j.195$
10000	$13.0+j50$	$.0195-j.075$	$20.0195+j99.925$	$.0193-j.096$

F *newtons*	V *m/s*	P *watts*	η *%*
$0.982 -j.0534$	$0.0688+j.0197$	0.0667	1.36
$.960 -j.149$	$.0738-j.0115$	.0726	1.51
$.796 -j.390$	$.0454-j.0457$	.0539	1.36
$.0386-j.192$	$-.00341-j.00166$	.000187	.097

This current flowing in the voice coil gives rise to a mechanic force acting on the diaphragm:

$$F = \mathfrak{A} I \ \textit{newtons.}$$

The diaphragm, vibrating in a vacuum, dissipates no energy by its motion. The power supplied to the circuit is absorbed in the electric resistance of the voice coil. (See Table III.)

The computations for this case are carried through in the same manner as those delineated in case Ia except that the mechanic impedance of the vibrating diaphragm contains a dissipative term accounting for the tendency of the system to supply acoustic energy to the surrounding medium, air. The magnitude of this mechanic or acoustic resistance is calculated from:

$$R = \rho c a \ \textit{mech. ohms (kg/s)},$$

where ρ = density of air = 1.205 kg/m^3 at 1 *atmos.*, 20°C,
c = velocity of propagation of sound in air,
= 344.0 m/s at 1 *atmos.*, 20°C,
a = area of diaphragm = 0.0314 m^2,
whence R = 13.0 *mech. ohms (kg/s)*.

The power dissipated in the mechanic system in this case may be determined from:

$$P = V^2 R \ \textit{watts.}$$

This power is supplied to the air as acoustic power. (Table IV.)

The computations for this case parallel those of case Ib with the exception noted above that the mechanic impedance of the system is altered by the air load on the diaphragm.

The efficiency of transfer of electric power to acoustic power is the ratio of acoustic power (= mechanic power dissipation for this case of frictionless supports) to electric power input to the voice coil and is given in the column headed η in Table V.